More on Tanaka-Tagoshi Parametrization of post-1PN Spin-Free Gravitational Wave Chirps: Equispaced and Cardinal Interpolated Lattices

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The conclusions obtained in gr-qc/0101067 are shown to be valid also if the full 2.5PN expansion of the chirp phase is used.

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In a recent paper [1] we shew that the spin-free gravitational wave templates parametrization introduced by Tanaka and Tagoshi in [2] is effective to set up *uniformly spaced* post-1PN template lattices subject to a given minimal match constraint. Lattice-uniformity makes cardinal interpolation of the match [3], [4] in the $2D$ Tagoshi-Tanaka parameter space straightforward, yielding a reduction in the 2PN template density and total number by a factor ~ 4 at $\Gamma = 0.97$.

In [1] the 2PN approximant of the (spectral, spin-free) chirp phase has been used. On the other hand, in [2] the 2.5PN phase (originated in [5]) has been used, which contains (besides an irrelevant f-independent term, which is absorbed in the unknown coalescency phase ϕ_C) the additional term:

$$\frac{\pi}{128\eta}\left(\frac{38645}{252}+5\eta\right)\cdot\log(f/f_0), \tag{0.1}$$

where $\eta = m_1 m_2/m^2$ is the symmetric mass-ratio, and f_0 is a suitable scaling frequency [6]. It has been pointed out [5] that the 2.5PN phase agrees, in the limit $\eta \to 0$, with the one obtained by Tagoshi and Sasaki in the (perturbative) test mass-limit solution of the spin-free relativistic two-body problem [7]. While this might be a good reason to include the 2.5PN term (0.1) in the template phase, it does *not* result into better overlaps with exact (numerically computed) waveforms. Indeed, as shown in [8], 2.5PN templates yield generally *poorer* overlaps (and larger biases) as compared to 2PN, in view of the peculiar (oscillating) behaviour of standard PN approximants [8].

However, the conceptual question is posed whether the results in [1] still hold if the 2.5PN logarithmic term (0.1) is included in the template phase. The purpose of this note is to show that the main conclusions in [1] do apply in this case too, though a number of differences are noted, which are summarized below.

The major difference, after the inclusion of the 2.5PN logarithmic term (0.1) in the phase, is a moderate increase in the area of the Tanaka-Tagoshi (flattened) manifold $\mathcal{T}$, yielding a correspondingly larger number of templates, as seen by comparing Fig. 1 and Table I of this note to Fig. 3 and Table III of [1], respectively.

The gaussian curvature of the spin-free manifold retains the same order of magnitude, as seen by comparing Fig. 2 of this note to Fig.s 1 and 2 of [1].

The main relevant conclusions in [1] do *not* change. As shown in [1], the minimal match degradation due to using the (flattened) Tanaka-Tagoshi (spin-free) parameter-space manifold in place of the (curved) true one, can be gauged in terms of the quantity η defined in Eq. (III.6) of [1]. Even using the 2.5PN phase, the quantity η is still of the same order of magnitude as the corresponding one computed in [1]. This is seen by comparing Fig. 3 of this note to Fig.s 5 and 6 of [1].

Acknowledgements

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Antenna	Simplex area $[sec^2]$	No. of templates at $\Gamma = 0.97$
TAMA300	10266	$1.71 \cdot 10^5$
GEO600	57093	$9.51 \cdot 10^5$
LIGO-I	24732	$4.12 \cdot 10^5$
VIRGO	776392	$1.29 \cdot 10^7$

Table I - 2.5PN flat simplex area and number of templates at $\Gamma = .97$,
for $0.2M_\odot \le m_1 \le m_2 \le 10M_\odot$.

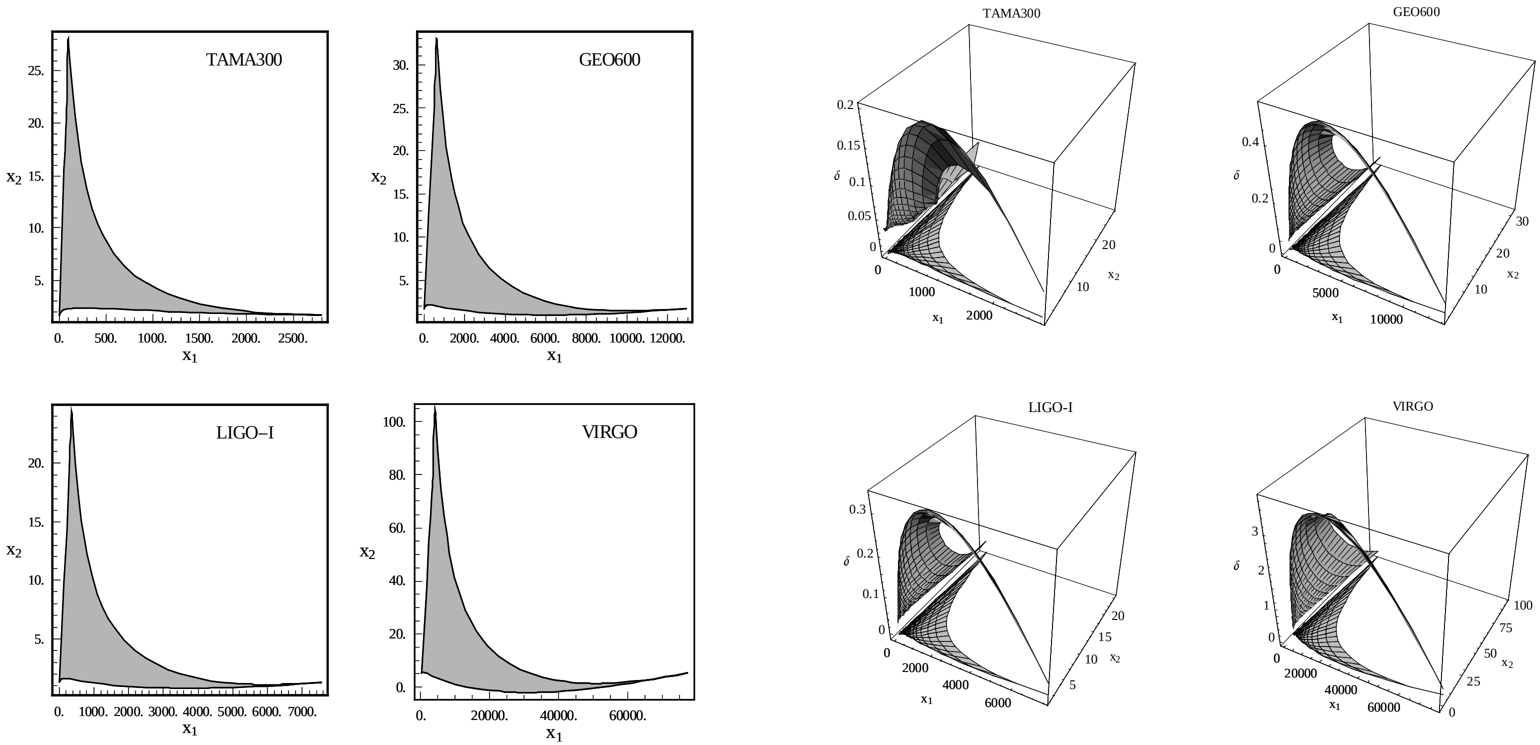

Fig 1 – Left: the 2.5PN flat simplexes corresponding to $0.2\ M_{\odot} \le m_1 \le m_2 \le 10\ M_{\odot}$ for TAMA300, GEO600, LIGO-1 and VIRGO; right: the euclidean distance between the 2.5PN manifolds $\mathcal{P}$ and $\mathcal{T}$ as a function of (x_1,x_2) for TAMA300, GEO600, LIGO-I and VIRGO.

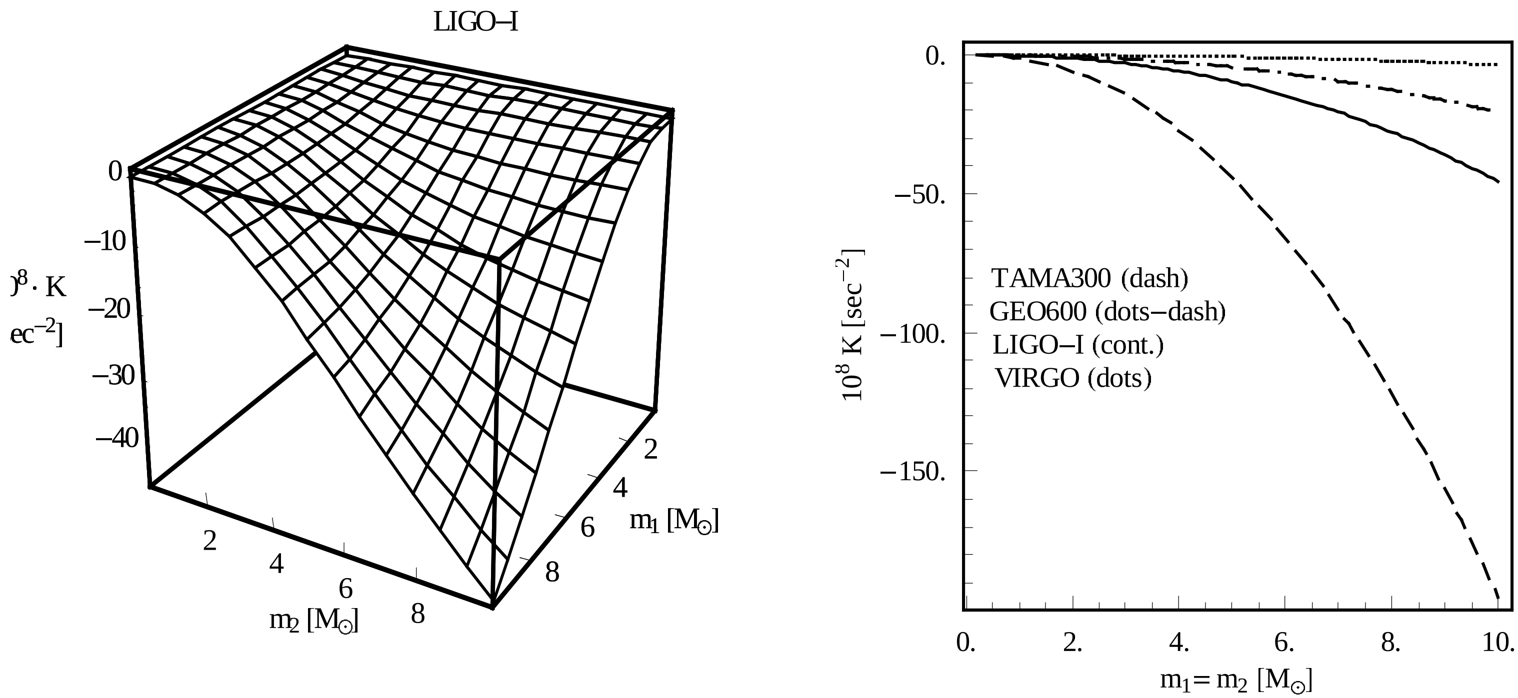

Fig. 2 – Left: gaussian curvature K [sec^{-2}] of spin-free 2.5PN parameter space manifold vs. m_1, m_2 for LIGO-1; right: the same quantity vs. m_1=m_2 (worst case) for TAMA300, GEO600, LIGO-1 and VIRGO.

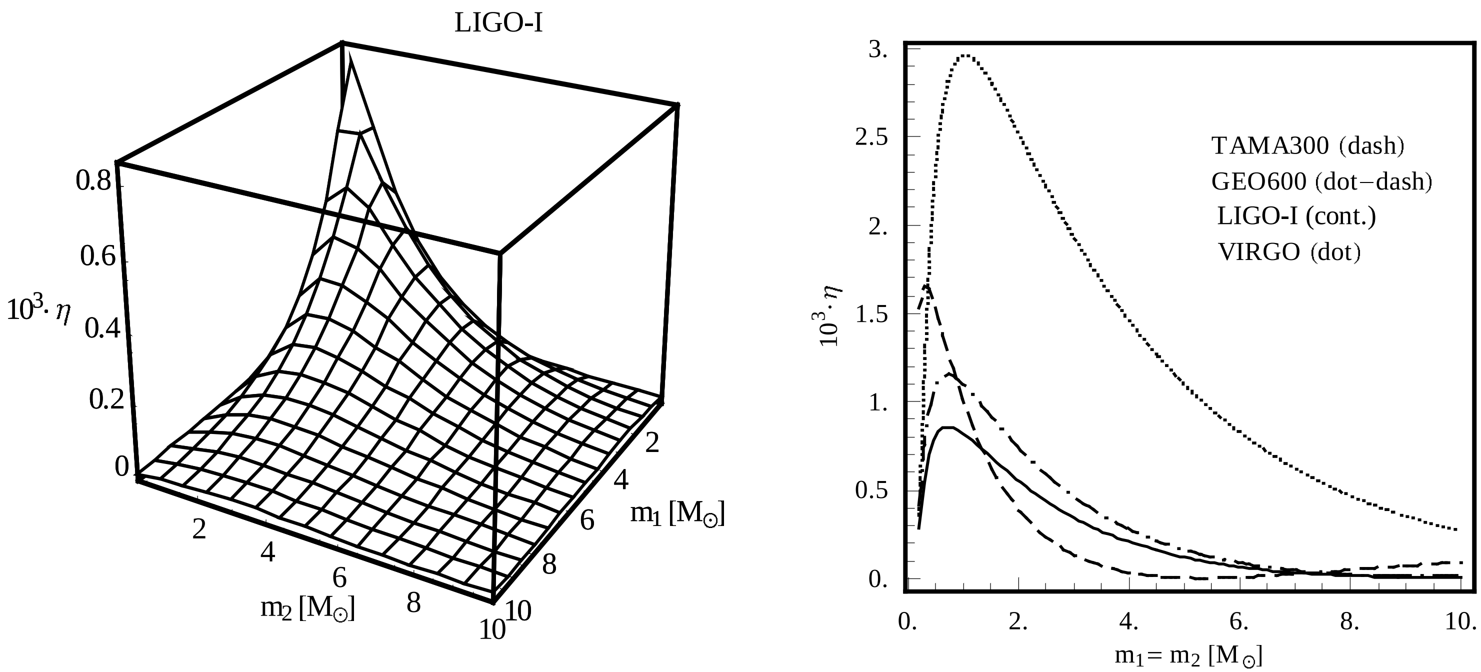

Fig 3 – Left: the 2.5PN quantity η (eq.III.6 of [1]) vs. m_1, m_2 for LIGO-I; right: the same quantity vs. m_1=m_2 (worst case) for TAMA300, GEO600, LIGO-I and VIRGO.